Synchronization of Hyperchaotic Memristor-Based Chua's Circuits

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Abstract This paper further investigates the problem of synchronization of hyperchaotic memristor-based Chua's circuits. An active control method is employed to design a controller to achieve the global synchronization of two identical memristor-based systems. Based on Lyapunov stability theory, a sufficient condition is given to guarantee the stability of the synchronization error system.

Keywords Memristor-based systems • Synchronization • Chua's circuit

1 Introduction

The existence of the memristor as the fourth fundamental circuit element included along with the resistor, capacitor, and inductor was predicated by Chua in 1971 [1]. Until 2008, the Hewlett-Packard (HP) research team announced that they had realized a prototype of memristor-based on nanotechnology [2]. Many researchers focus on the memristor because of its potential applications in programmable logic, signal processing, neural networks, control systems, reconfigurable computing, Brain-computer interfaces, and RFID [3–9].

Recently, the research on circuits based on memristor is becoming a focal topic [10–19]. Itoh and Chua presented a fourth-order memristor-based Chua's oscillator by replacing Chua's diode with an active two-terminal circuit consisting of a conductance and a flux-controlled memristor [10]. Pershin and Di Ventra introduced an approach to use memristors in programmable analog circuits [11]. Rak and

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Cserey presented a new simulation program with integrated circuit emphasis macro-model of the recently physically implemented memristor [12]. Petras presented fractional-order memristor-based Chua's circuit [13]. The hyperchaotic behavior in memristor-based Chua's circuit is performed with the help of nonlinear tools [19]. In this letter, we will study the problem of synchronization of memristor-based Chua's systems. Based on the feedback control method, we design a controller to guarantee the exponentially stability of the synchronization error system.

The rest of the paper is organized as follows. In Sect. 2, a memristor-based system is introduced. And using feedback control method, a general convergence criterion for stabilization of synchronization error system is established. Conclusions are finally drawn in Sect. 3.

2 Problem Formulation and Preliminaries

Referring to [19], Andrew L. Fitch proposed a circuit by adding an inductor in parallel with conductance—G and fourth-order memristor-based canonical oscillato. The equations for the circuit are described by

$$\begin{cases} \frac{dq_{l2}(t)}{dt} = \frac{1}{L_2}\phi_{c2}(t), \\ \frac{d\phi_{c2}(t)}{dt} = \frac{1}{C_2}\left(-q_{l2}(t) - G\phi_{c2}(t) - q_{l1}(t)\right), \\ \frac{dq_{l1}(t)}{dt} = \frac{1}{L_1}\left(\phi_{c2}(t) - q_{l2}(t)R - \phi_{c1}(t)\right), \\ \frac{d\phi_{c1}(t)}{dt} = \frac{1}{C_1}\left(-q_{l1}(t)R - a\phi_{c1}(t) - b{\phi_{c1}}^3(t)\right). \end{cases} \tag{1}$$

The system Eq. (1) can be reorganized by

$$\begin{cases} \frac{d\phi_{c1}(t)}{dt} = -\tau v_1(t), \\ \frac{dv_1(t)}{dt} = \frac{1}{C_1}\left(i_{L1}(t) - W\left(\phi_{c1}(t)\right)v_1(t)\right), \\ \frac{dv_2(t)}{dt} = \frac{1}{C_2}\left(Gv_2(t) - i_{L1}(t) - i_{L2}(t)\right), \\ \frac{di_{L1}(t)}{dt} = \frac{1}{L_1}\left(v_2(t) - v_1(t) - Ri_{L1}(t)\right) \\ \frac{di_{L2}(t)}{dt} = \frac{1}{L_2}v_2(t). \end{cases} \tag{2}$$

where τ is an integration constant which introduced to rescale the values of voltage into practical range, and $W(\phi_{c1}(t)) = a + 3b{\phi_{c1}}^2(t)$.

Letting $x_1(t) = \phi_{c1}(t)$, $x_2(t) = v_1(t)$, $x_3(t) = v_2(t)$, $x_4(t) = i_{L1}(t)$, $x_5(t) = i_{L2}(t)$, system (2) can be further rewritten as

$$\begin{cases} \dot{x}_1(t) = -\tau x_2(t), \\ \dot{x}_2(t) = \frac{1}{C_1}\left(x_4(t) - ax_2(t) - 3b{x_1}^2(t)x_2(t)\right), \\ \dot{x}_3(t) = \frac{1}{C_2}\left(Gx_3(t) - x_4(t) - x_5(t)\right), \\ \dot{x}_4(t) = \frac{1}{L_1}\left(x_2(t) - x_3(t) - Rx_4(t)\right), \\ \dot{x}_5(t) = \frac{1}{L_2}x_3(t). \end{cases} \tag{3}$$

When $L_1 = 10mH$, $L_2 = 60mH$, $C_1 = 6.8nF$, $C_2 = 15nF$, $G = 0.0005S$, $a = 0.00067$, $b = 0.000029$, $R = 65\Omega$, $\tau = 26,000$, existence of hyperchaos.

The system (3) can be rewritten with linear part and nonlinear part as follows:

$$\dot{x}(t) = Ax(t) + f\left(x(t)\right), \tag{4}$$

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t))^T$,

$$A = \begin{bmatrix} 0 & -\tau & 0 & 0 & 0 \\ 0 & \frac{-a}{C_1} & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & \frac{G}{C_1} & \frac{-1}{C_2} & \frac{-1}{C_2} \\ 0 & \frac{1}{L_1} & \frac{-1}{L_1} & \frac{-R}{L_1} & 0 \\ 0 & 0 & \frac{-1}{L_2} & 0 & 0 \end{bmatrix}, \quad A = \begin{pmatrix} 0 & -\tau & 0 & 0 & 0 \\ 0 & -a/C_1 & 0 & 1/C_1 & 0 \\ 0 & 0 & G/C_2 & -1/C_2 & -1/C_2 \\ 0 & 1/L_1 & -1/L_1 & -R/L_1 & 0 \\ 0 & 0 & 1/L_2 & 0 & 0 \end{pmatrix}$$

and

$$f\left(x(t)\right) = \begin{pmatrix} 0 \\ -3bx_1{}^2 x_2/C_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

As for vector function $f(x)$, assume that for any $x, y \in \Omega$ we have

$$\left| f_i(x) - f_i(y) \right| \le L_{\max} \left| x - y \right|, \; i = 1, \; 2, \; 3, \; 4 \tag{5}$$

The above condition is considered as the uniform Lipszchitz condition, and $L_{\max} > 0$ refers to the uniform Lipschitz constant.

We construct the response system as below:

$$\dot{y}(t) = Ay(t) + f(y(t)) + u(t) \tag{6}$$

where $y(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^T$ is the response state, $u(t)$ is the control gain defined by:

$$u(t) = k(y(t) - x(t)).$$

where $\tau > 0$ is the propagation delay, k denotes control strength. Let $e(t) = y(t) - x(t)$ be the synchronization error between the systems (2) and (3), then yields the error system

$$\begin{aligned} \dot{e}(t) &= \dot{y}(t) - \dot{x}(t) \\ &= Ae(t) + f(y) - f\left(x(t)\right) + u(t) \\ &= Ae(t) + f(y) - f\left(x(t)\right) + ke(t) \end{aligned} \tag{7}$$

We now state our main results.

Theorem 1. *Suppose that there exist positive constants* s_1, g_1 *such that* $A + A^T + 2kI + s_1 I + {s_1}^{-1} {L_{\max}}^2 + g_1 I \leq 0$,

Then, the synchronization error system (7) *is globally exponentially stable, and the systems* (4) *and* (6) *are globally exponentially synchronized.*

Proof. Choose the Lyapunov function as follows

$$V(t) = e(t)^T e(t). \tag{8}$$

Then the differentiation of V along trajectories of (7) is

$$\begin{aligned}
\dot{V}(t) &= e(t)^T \dot{e}(t) + \dot{e}(t)^T e(t) \\
&= e(t)^T \left[Ae(t) + f(y(t)) - f(x(t)) + ke(t)\right] + \\
&\quad \left[Ae(t) + f(y(t)) - f(x(t)) + ke(t)\right]^T e(t) \\
&\leq e(t)^T \left[A + A^T + 2kI\right] e(t) + s_1 e(t)^T e(t) \\
&\quad + {s_1}^{-1} \left[f(y(t)) - f(x(t))\right]^T \left[f(y(t)) - f(x(t))\right] \\
&\leq e(t)^T \left[A + A^T + 2kI + s_1 I\right] e(t) + {s_1}^{-1} {L_{\max}}^2 e(t)^T e(t) \\
&= e(t)^T \left[A + A^T + 2kI + s_1 I + {s_1}^{-1} {L_{\max}}^2 + g_1 I\right] e(t) - g_1 e(t)^T e(t) \\
&\leq -g_1 e(t)^T e(t) \\
&= -g_1 V(t)
\end{aligned}$$

According to Lyapunov theory, the inequality $\dot{V}(t) \leq -g_1 V(t)$ indicate $V(t)$ converges to zero exponentially. Furthermore, we can conclude that the synchronization error systems $e(t)$ converges to zero globally and exponentially with a rate g_1, and the synchronization between with system (4) and system (6) can be obtained. This completes the proof. □

3 Conclusions

In this paper, the synchronization problem of memristor-based chaotic system has been discussed. A feedback controller was designed to stabilize the synchronization error system globally exponentially.

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