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Data-driven Predictive Control for Continuous-time Linear Parameter Varying Systems with Application to Wind Turbine

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Abstract: A new data-driven predictive control method based on subspace identification for continuous-time linear parameter varying (LPV) systems is presented in this paper. It is developed by reformulating the continuous-time LPV system which utilizes Laguerre filters to obtain the subspace prediction of output. The subspace predictors are derived by QR decomposition of input-output and Laguerre matrices obtained by input-output data. The predictors are then applied to design the model predictive controller. It is shown that the integrated action is incorporated in the control effect to eliminate the steady-state offset. We control the continuous-time LPV systems to obtain the attractive performance with the proposed data-driven predictive control method. The proposed controller is applied to a wind turbine to verify its effectiveness and feasibility.

Keywords: Continuous-time, data-driven approach, linear parameter varying systems, model predictive control, subspace identification.

1. INTRODUCTION

The linear parameter varying(LPV) systems become more attractive than before and they can be seen as a particular type of time-varying system. The LPV models can be used to approximate nonlinear systems with much lower system order but higher precision than linear model approximations. Moreover, the LPV models allow the extension of linear design techniques to nonlinear systems [1]. Because of these advantages, there are many researches in LPV systems [2–4]. A number of LPV model applications have emerged, including wind turbines [5], biomedical applications [6], and leakage detection [7]. As industrial processes grow increasingly complex, continuous-time systems have grown increasingly common (e.g., continuous rolling process, aircraft flight processes [8, 9]) and are difficult to model due to the high level of complexity, necessitating continuous-time identification methodology. Generally, the continuous-time identification falls into two distinguish categories: The indirect approach and direct approach [10]. The indirect approach basically view the situation at two points: First by using a non-parametric model like impulse response, step response or frequency response function. Second step is to estimate continuous time parameters from the estimated discrete time model. The drawbacks of the indirect approach are that the sampling time is difficult to select and zero pole conversion is not consistent. In contrast, the direct approach often approximates the derivative operator that is associated with input and output signal using a filter to identify the continuous-time models directly. The direct approach shows superior performance and is, accordingly, a more popular research focus [11].

Subspace identification is one available system identification algorithm for state-space modeling, through which workers engaged in automation do not need to perform tedious mechanism modeling and the accurate state-space model can be obtained once there is enough process input-output data [12, 13]. Subspace identification for LPV systems has obtained considerable attention. Vincent and Verhaegen [14, 15] presented subspace identification for multivariable LPV state-space systems with affine parameter dependence and kernel methods for reducing dimensions in LPV systems. In [16], the periodic scheduling sequence is used from LTI subspace identification to determine the column space of the time-varying observability matrices. The open- and closed-loop data can be well solved through subspace identification of LPV systems in [17]. The subspace identification of continuous-time systems has been studied in a number of contributions. In [18] an approach for system identification of continuous-time stochastic state space models from random input-output continuous data was presented. The approach is based on the introduction of random distribution theory in

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describing (higher) time derivatives of stochastic processes, and the input-output algebraic relationship is derived which is treated in the time-domain. Wu et al. [19] solved the continuous-time identification for errors-in-variables based on the linear filter method and principal component analysis. Bergamasco and Lovera [20] dealt with the problem of continuous-time model identification and presented two subspace-based algorithms capable of dealing with data generated by systems operating in closed loop. These methods only solve the identification problem, but in this study, we obtain the appropriate subspace predictors using continuous-time subspace identification for LPV systems to design an innovative predictive controller.

Model predictive control (MPC) is a powerful model based control technique, which explicitly optimizes the overall performance of a system to be controlled [21]. There are several attractive features of MPC; e.g., it handles multivariable control problems naturally, it considers input and output constraints, and it adapts structural changes [22]. By combining the merits of subspace identification and MPC, data-driven predictive control was formed [23–26]. It is a powerful technique which exploits subspace identification to design continuous-time model predictive controllers in LPV systems. Importantly, this is the first paper to our knowledge to report a data-driven predictive control for continuous-time LPV systems.

The primary contribution of this study is that we are able to obtain key subspace predictors to design the model predictive controller in a continuous-time LPV system. The system is first transformed into the Laguerre form to obtain the subspace prediction of the output via recursive substitution, then through RQ factorization, the subspace predictors are obtained from the R matrix as-derived from input-output and Laguerre matrices. The incremental form of the cost function is then constructed in MPC and the subspace predictors are incorporated to obtain the control input. A seven-degrees-of-freedom wind turbine model is used to verify the performance of the proposed control method; excellent control performance is observed as evidenced by simulation results.

The outline of the paper is arranged as follows. We start in Section 2 with the subspace prediction of output. In Section 3, we give the data-driven predictive control method. In Section 4 the simulation example is presented that shows the potential of the proposed method. Section 5 ends with the conclusions.

2. SUBSPACE PREDICTION OF OUTPUT

Consider the LPV system described by continuous-time form:

$$x(t+1) = A(t)x(t) + B(t)u(t) + w(t), \tag{1}$$

$$y(t) = C(t)x(t) + D(t)u(t) + v(t), \tag{2}$$

where $u(t) \in \mathbb{R}^l$, $y(t) \in \mathbb{R}^m$, and $x(t) \in \mathbb{R}^n$ are input, output, and state vectors respectively. The time varying system matrix is now given by

$$A(t) = \sum_{i=1}^{m} A^{(i)} \mu^{(i)}(t) \tag{3}$$

and $B(t)$, $C(t)$, and $D(t)$ are similar to $A(t)$. The matrices $A^{(i)} \in \mathbb{R}^{n\times n}$, $B^{(i)} \in \mathbb{R}^{n\times l}$, $C^{(i)} \in \mathbb{R}^{m\times n}$, $D^{(i)} \in \mathbb{R}^{m\times l}$. The model weights $\mu_t^{(i)} \in \mathbb{R}$. $w(t) \in \mathbb{R}^n$ and $v(t) \in \mathbb{R}^m$ are zero-mean white Gaussian sequences with covariance matrix:

$$\mathrm{E}\left[\begin{pmatrix} w(t) \\ v(t) \end{pmatrix} \begin{pmatrix} w(t)^{\mathrm{T}} & v(t)^{\mathrm{T}} \end{pmatrix}\right] = \begin{bmatrix} Q & S \\ S^{\mathrm{T}} & R \end{bmatrix} \delta_{ij}, \tag{4}$$

where δ_{ij} is Kronecker delta.

2.1. Continuous-time prediction of output

The i-th continuous-time Laguerre filter is given by

$$L_i(s) = \sqrt{2p}\frac{(s-a)^i}{(s+a)^{i+1}}, \tag{5}$$

where $a > 0$ is the scaling factor to ensure that the filters are stable. Define a w-operator that corresponds to the all-pass Laguerre filter which has the form

$$w(s) = \frac{s-a}{s+a}. \tag{6}$$

Through multiplication, the system can be transformed as follows [27],

$$[w\hat{x}](t) = A_w\hat{x}(t) + B_w[l_0u](t) + [l_0w_w](t) + K_1x_0l_0(t), \tag{7}$$

$$[l_0y](t) = C_w\hat{x}(t) + D_w[l_0u](t) + [l_0v_w](t) + K_2x_0l_0(t), \tag{8}$$

where

$$\begin{aligned}
A_w &= (A+aI)^{-1}(A-aI),\\
B_w &= (A+aI)^{-1}B,\\
C_w &= 2aC(A+aI)^{-1},\\
D_w &= D - C(A+aI)^{-1}B,\\
K_1 &= (A+aI)^{-1},\\
K_2 &= C(A+aI)^{-1},\\
w_w(t) &= (A+aI)^{-1}w(t),\\
v_w(t) &= v(t) - C(A+aI)^{-1}w(t),
\end{aligned} \tag{9}$$

and x_0 is the initial state of the original continuous-time system.

The subspace prediction output of the continuous-time system can be derived by recursive substitution of (7)-(8):

$$Y_{i,j}(t) = \Gamma_j x(t) + H_j U_{i,j}(t) + H_j^s W_{i,j}(t) + V_{i,j}(t) + F_j \Psi_{i,j}(t) \tag{10}$$

where

$$Y_{i,j}(t) = \begin{bmatrix} [l_iy](t) \\ [l_{i+1}y](t) \\ \vdots \\ [l_{i+j-1}y](t) \end{bmatrix}, \Gamma_j = \begin{bmatrix} C_w \\ C_wA_w \\ \vdots \\ C_wA_w^{j-1} \end{bmatrix},$$

$$H_j = \begin{bmatrix} D_w & 0 & \cdots & 0 \\ C_w B_w & D_w & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_w A_w^{j-2} B_w & \cdots & C_w B_w & D_w \end{bmatrix},$$

$$H_j^s = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_w & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_w A_w^{j-2} & \cdots & C_w & 0 \end{bmatrix},$$

$$U_{i,j}(t) = \begin{bmatrix} [l_i u](t) \\ [l_{i+1} u](t) \\ \vdots \\ [l_{i+j-1} u](t) \end{bmatrix}, \; W_{i,j}(t) = \begin{bmatrix} [l_i w_w](t) \\ [l_{i+1} w_w](t) \\ \vdots \\ [l_{i+j-1} w_w](t) \end{bmatrix},$$

$$V_{i,j}(t) = \begin{bmatrix} [l_i v_w](t) \\ [l_{i+1} v_w](t) \\ \vdots \\ [l_{i+j-1} v_w](t) \end{bmatrix}, \; \Psi_{i,j}(t) = \begin{bmatrix} l_i(t) \\ l_{i+1}(t) \\ \vdots \\ l_{i+j-1}(t) \end{bmatrix},$$

$$F_j = \begin{bmatrix} K_2 x_0 & 0 & \cdots & 0 \\ C_w K_1 x_0 & K_2 x_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_w A_w^{j-2} K_1 x_0 & \cdots & C_w K_1 x_0 & K_2 x_0 \end{bmatrix}.$$

The above result is taken from the literature [20]. We next derive the subspace prediction for LPV systems through the proposed method for the sake of comparison.

2.2. Continuous-time prediction of output for LPV systems

Define the output vector y_t^d as

$$y_t^d = [y_t^{\mathrm{T}} \; y_{t+1}^{\mathrm{T}} \; \cdots \; y_{t+d-1}^{\mathrm{T}}]^{\mathrm{T}} \tag{11}$$

and the input vector u_t^d, the noise vector w_t^d and v_t^d are similar to y_t^d where d is defined as the window size. Define the transition matrix $\Phi_A(t,j) \in \mathbb{R}^{n\times n}$ for $t > j$:

$$\Phi_A(t,j) = A_{t-1} A_{t-2} \cdots A_j, \tag{12}$$

where $\Phi_A(t,t) = I_n$, I is the identity matrix. The subspace prediction of output with the LPV system can be derived by recursive substitution of Eqs. (1)-(2):

$$y_t^d = \Gamma_t^d x_t + H_t^d u_t^d + \varepsilon_t^d w_t^d + v_t^d \tag{13}$$

with

$$\Gamma_t^d = \begin{bmatrix} C_t \\ C_{t+1}\Phi_A(t+1,t) \\ \vdots \\ C_{t+d-1}\Phi_A(t+d-1,t) \end{bmatrix},$$

$$H_t^d = \begin{bmatrix} D_t & 0 & \cdots & 0 \\ h_{t,2,1}^d & D_{t+1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ h_{t,d,1}^d & \cdots & h_{t,d,d-1}^d & D_{t+d-1} \end{bmatrix},$$

$$\varepsilon_t^d = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \varepsilon_{t,2,1}^d & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \varepsilon_{t,d,1}^d & \cdots & \varepsilon_{t,d,d-1}^d & 0 \end{bmatrix},$$

where $h_{t,i,j}^d = C_{t+i-1}\Phi_A(t+i-1,t+j)B_{t+j-1}$, $\varepsilon_{t,i,j}^d = C_{t+i-1}\Phi_A(t+i-1,t+j)$ with $i = 2,\cdots,d$, $j = 1,\cdots,d-1$, and $i > j$.

The system stability analysis can refer to "2.3 Statistical framework" of paper [14].

To use the continuous-time subspace prediction of output to design model predictive controller, define the period is p and N samples in system. The transition matrix $\overline{\Phi}_A(t,j) \in \mathbb{R}n \times n$ for $t > j$ is

$$\overline{\Phi}_A(t,j) = A_{w(t-1)} A_{w(t-2)} \cdots A_{wj} \tag{14}$$

and $\overline{h}_{t,i,j}^d = C_{w(t+i-1)}\overline{\Phi}_A(t+i-1,t+j)B_{w(t+j-1)}$, $\overline{\varepsilon}_{t,i,j}^d = C_{w(t+i-1)}\overline{\Phi}_A(t+i-1,t+j)$.

Equation (13) can be transformed as

$$Y_{i,d,N}^t = \overline{\Gamma}_t^d X_{i,N}^t + \overline{H}_t^d U_{i,d,N}^t + \overline{\varepsilon}_t^d W_{i,d,N}^t + V_{i,d,N}^t + F_t^d \Psi_{i,d,N}^t, \tag{15}$$

where

$$Y_{i,d,N}^t = [l_i y_{t+ip}^d \;\; l_{i+1} y_{t+(i+1)p}^d \;\; \cdots \;\; l_{i+N-1} y_{t+(i+N-1)p}^d],$$
$$U_{i,d,N}^t = [l_i u_{t+ip}^d \;\; l_{i+1} u_{t+(i+1)p}^d \;\; \cdots \;\; l_{i+N-1} u_{t+(i+N-1)p}^d],$$
$$W_{i,d,N}^t = [l_i w_{w(t+ip)}^d \;\; l_{i+1} w_{w[t+(i+1)]p}^d \;\; \cdots \;\; l_{i+N-1} w_{w[t+(i+N-1)p]}^d],$$
$$V_{i,d,N}^t = [l_i v_{w(t+ip)}^d \;\; l_{i+1} v_{w[t+(i+1)]p}^d \;\; \cdots \;\; l_{i+N-1} v_{w[t+(i+N-1)p]}^d],$$
$$X_{i,N}^t = [x_{t+ip} \;\; x_{t+(i+1)p} \;\; \cdots \;\; x_{t+(i+N-1)p}],$$
$$\Psi_{i,N}^t = [l_{t+ip}^d \;\; l_{t+(i+1)p}^d \;\; \cdots \;\; l_{t+(i+N-1)p}^d],$$

$$\overline{\Gamma}_t^d = \begin{bmatrix} C_w t \\ C_{w(t+1)}\overline{\Phi}_A(t+1,t) \\ \vdots \\ C_{w(t+d-1)}\overline{\Phi}_A(t+d-1,t) \end{bmatrix},$$

$$\overline{H}_t^d = \begin{bmatrix} D_w t & 0 & \cdots & 0 \\ \overline{h}_{t,2,1}^d & D_{w(t+1)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \overline{h}_{t,d,1}^d & \cdots & \overline{h}_{t,d,d-1}^d & D_{w(t+d-1)} \end{bmatrix},$$

$$\overline{\varepsilon}_t^d = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \overline{\varepsilon}_{t,2,1}^d & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \overline{\varepsilon}_{t,d,1}^d & \cdots & \overline{\varepsilon}_{t,d,d-1}^d & 0 \end{bmatrix},$$

$$F_t^d = \begin{bmatrix} K_2 x_0 & 0 & \cdots & 0 \\ C_w t K_1 x_0 & K_2 x_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \Delta_1 & \cdots & \Delta_2 & K_2 x_0 \end{bmatrix}.$$

where $\Delta_1 = C_{w(t+d-2)}\overline{\Phi}_A(t+d-2,t)K_1x_0$, $\Delta_2 = C_{w(t+d-2)}K_1x_0$.

3. DATA-DRIVEN PREDICTIVE CONTROL

In this section, we first derive the applicable subspace predictors for continuous-time LPV systems using the subspace prediction discussed in Section 2.2.

Construct the following instrumental variable matrix W_N :

$$W_N^{t+p-d} = \begin{bmatrix} U_{0,d,N}^{t+p-d} \\ Y_{0,d,N}^{t+p-d} \end{bmatrix}, \tag{16}$$

where

$$U_{0,d,N}^{t+p-d} = \begin{bmatrix} l_0u_{t+p-d} & l_0u_{t+2p-d} & \cdots & l_0u_{t+N-d} \\ l_1u_{t+p-d+1} & l_1u_{t+2p-d+1} & \cdots & l_1u_{t+N-d+1} \\ \vdots & \ddots & \ddots & \vdots \\ l_{d-1}u_{t+p-1} & l_{d-1}u_{t+2p-1} & \cdots & l_{d-1}u_{t+N-1} \end{bmatrix},$$

$$Y_{0,d,N}^{t+p-d} = \begin{bmatrix} l_0y_{t+p-d} & l_0y_{t+2p-d} & \cdots & l_0y_{t+N-d} \\ l_1y_{t+p-d+1} & l_1y_{t+2p-d+1} & \cdots & l_1y_{t+N-d+1} \\ \vdots & \ddots & \ddots & \vdots \\ l_{d-1}y_{t+p-1} & l_{d-1}y_{t+2p-1} & \cdots & l_{d-1}y_{t+N-1} \end{bmatrix}.$$

The $U_{1,d,N}^t$ and $Y_{1,d,N}^t$ are represented similarly with $U_{0,d,N}^{t+p-d}$ and $Y_{0,d,N}^{t+p-d}$.

The instrumental variable matrix $\Psi_{1,d,N}^t$ is as:

$$\Psi_{1,d,N}^t = \begin{bmatrix} l_{t+p} & l_{t+2p} & \cdots & l_{t+N} \\ l_{t+p+1} & l_{t+2p+1} & \cdots & l_{t+N+1} \\ \vdots & \ddots & \ddots & \vdots \\ l_{t+p+d-1} & l_{t+2p+d-1} & \cdots & l_{t+N+d-1} \end{bmatrix}. \tag{17}$$

Take the RQ factorization:

$$\begin{aligned} \begin{bmatrix} \Psi_{1,d,N}^t \\ W_N^{t+p-d} \\ U_{1,d,N}^t \\ Y_{1,d,N}^t \end{bmatrix} &= R^{\mathrm{T}}Q^{\mathrm{T}} \\ &= \begin{bmatrix} R_{11}^t & 0 & 0 & 0 \\ R_{21}^t & R_{22}^t & 0 & 0 \\ R_{31}^t & R_{32}^t & R_{33}^t & 0 \\ R_{41}^t & R_{42}^t & R_{43}^t & R_{44}^t \end{bmatrix} \begin{bmatrix} Q_1^{\mathrm{T}} \\ Q_2^{\mathrm{T}} \\ Q_3^{\mathrm{T}} \\ Q_4^{\mathrm{T}} \end{bmatrix}. \end{aligned} \tag{18}$$

The optimal prediction $\hat{Y}_{1,d,N}^t$ can be found from the orthogonal projection of the row space of $Y_{1,d,N}^t$ onto the row space of the matrix $\begin{bmatrix} \Psi_{1,d,N}^t \\ W_N^{t+p-d} \\ U_{1,d,N}^t \end{bmatrix}$:

$$\hat{Y}_{1,d,N}^t = Y_{1,d,N}^t / \begin{bmatrix} \Psi_{1,d,N}^t \\ W_N^{t+p-d} \\ U_{1,d,N}^t \end{bmatrix}. \tag{19}$$

The optimal prediction $\hat{Y}_{1,d,N}^t$ also can be written as

$$\hat{Y}_{1,d,N}^t = L_w^t W_N^{t+p-d} + L_u^t U_{1,d,N}^t, \tag{20}$$

where L_w^t is the subspace predictor that corresponds to the past input-output data and L_u^t is the subspace predictor that corresponds to the future input data.

Through the implementation of the orthogonal projection, by letting

$$\begin{aligned} &\begin{bmatrix} L_w^t & L_u^t \end{bmatrix} \\ &= \begin{bmatrix} R_{41}^t & R_{42}^t & R_{43}^t \end{bmatrix} \begin{bmatrix} R_{21}^t & R_{22}^t & 0 \\ R_{31}^t & R_{32}^t & R_{33}^t \end{bmatrix}^{\dagger}, \end{aligned} \tag{21}$$

where superscript † represents the Moore-Penrose pseudo-inverse. We can get the L_w^t and L_u^t.

Next, consider an apparent incremental form of the cost function which has an integrated action to eliminate the steady-state offset [23].

$$\begin{aligned} J &= \sum_{k=1}^{N_2} (r_{t+k} - \hat{y}_{t+k|t})^2 + \sum_{j=1}^{N_u} \lambda (\Delta u_{t+j-1})^2 \\ &= (r_f - \hat{y}_f)^{\mathrm{T}}(r_f - \hat{y}_f) + \Delta u_f^{\mathrm{T}}(\lambda I)\Delta u_f, \end{aligned} \tag{22}$$

where N_2 and N_u are the prediction and control horizon respectively, r_{t+k} is the reference setpoint signal at the current time $t+k$, λ is the weighting on the control effort. The vector of the optimal prediction of the future outputs can be expressed in terms of the future inputs and current states as

$$\hat{y}_f = Fy_t + L_w^{\circ}(1:N_2m,:)\Delta w_p + S_{N_2,N_u}\Delta u_f, \tag{23}$$

where $\hat{y}_f = \begin{bmatrix} \hat{y}_{t+1} & \cdots & \hat{y}_{t+N_2} \end{bmatrix}^{\mathrm{T}}$, $F = \begin{bmatrix} I_m & \cdots & I_m \end{bmatrix}^{\mathrm{T}}$, $S_{N_2,N_u} = L_u(1:N_2m, 1:N_ul)\begin{bmatrix} I_l & 0 & \cdots & 0 \\ I_l & I_l & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_l & I_l & \cdots & I_l \end{bmatrix}$. L_w° is constructed from L_w^t as

$$\begin{aligned} &L_w^{\circ}(m(k-1)+1:mk,:) \\ &= \sum_{i=1}^{k} L_w^t(m(i-1)+1:mi,:), \end{aligned} \tag{24}$$

where $1 \le k \le N_2$. Substituting the optimal prediction of the future outputs in (23) into the cost function in (22), differentiate it with respect to Δu_f and control sequence can be obtained:

$$\Delta u_f = \left(S_{N_2,N_u}^{\mathrm{T}} S_{N_2,N_u} + \lambda I\right)^{-1} S_{N_2,N_u}^{\mathrm{T}} \Delta_3, \tag{25}$$

where $\Delta_3 = (r_f - Fy_t - L_w^{\circ}(1:N_2m,:)\Delta w_p)$.

At each time instance, only the first element of Δu_f is used for calculating the control input. Therefore the control input u_t is drawn as

$$u_t = u_{t-1} + \Delta u_t. \tag{26}$$

Table 1. The circuit framework of proposed method.

1. Construct the LPV system described by continuous-time form as (1)-(2).
2. Obtain the continuous-time subspace prediction of output with (10).
3. Get the continuous-time subspace prediction of output for LPV systems with (15).
4. Computer the subspace predictors L_w^t and L_u^t with (21).
5. Derive the vector of the optimal prediction $\hat{y}_f$ of model predictive controller with (23).
6. Implement the control input u using Eqs. (25)-(26).
7. At the next time, when new data arrives, the following control input can be calculated using above steps.

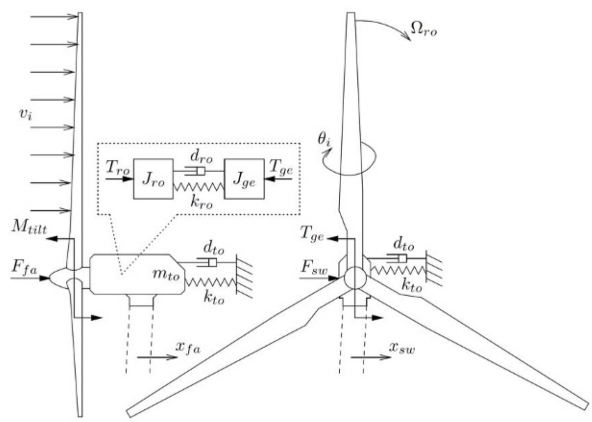

Fig. 1. The diagram of the wind turbine model.

At the next time sample, we measure the new input-output data and the new control input will be calculated using above procedure.

For the sake of clarity, the circuit framework of proposed method is summarized in Table 1.

4. SIMULATION EXAMPLE

The example is a wind turbine of seven degrees of freedom as described in [28, 29]. The model describes the rotational dynamics of a wind turbine around a particular operating point. The model contains degrees of freedom for one of the main rotation, two of first torsion mode of the drive train, two of the first fore-aft, and two of sideward bending mode of the tower. In the model, the blades are considered to be rigid. The diagram of the model is shown in Fig. 1.

The system has three rotor blades and can be linearized described in the following continuous-time LPV form:

$$\dot{x} = Ax + [B^{(1)} + \sum_{i=1}^{3} B^{(i+1)}\varphi^{(i)}]u + [F^{(1)} + \sum_{i=1}^{3} F^{(i+1)}\varphi^{(i)}]v, \tag{27}$$

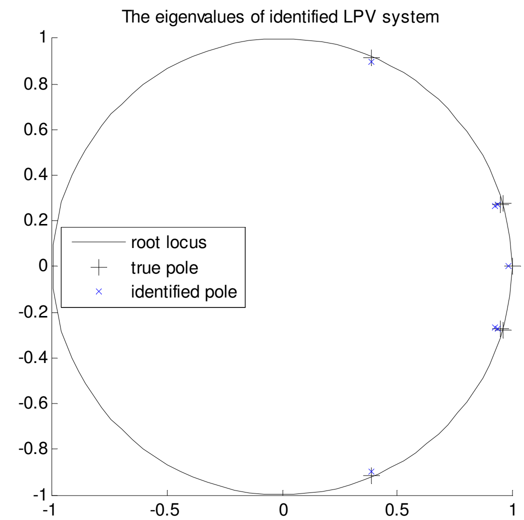

Fig. 2. The eigenvalues of $A^{(i)}$ matrices of the wind turbine.

$$y = [C^{(1)} + \sum_{i=1}^{3} C^{(i+1)}\varphi^{(i)}]x + Du + Gv. \tag{28}$$

The system input vector, output vector, state vector, and disturbance vector are given as: $u = [u_1 \quad u_2 \quad u_3 \quad u_4]^{\mathrm{T}} = [\sigma\theta_1 \quad \sigma\theta_2 \quad \sigma\theta_3 \quad \sigma T_{ge}]^{\mathrm{T}}$, $y = [y_1\ y_2\ y_3\ y_4\ y_5\ y_6]^{\mathrm{T}} = [\sigma\Omega_{ge}\ \dot{x}_{fa}\ \dot{x}_{sw}\ \sigma M_1\ \sigma M_2\ \sigma M_3]^{\mathrm{T}}$, $x = [\sigma\Omega_{ro}\ x_{fa}\ \dot{x}_{fa}\ x_{sw}\ \dot{x}_{sw}\ \varepsilon\ \dot{\varepsilon}]^{\mathrm{T}}$, $v = [\sigma v_1 \quad \sigma v_2 \quad \sigma v_3]^{\mathrm{T}}$, where $\sigma\theta_i$ is the pitch angle variation, σT_{ge} is the generator torque, $\sigma\Omega_{ge}$ is the variation in generator speed, σM_i is the blade root bending moment, $\sigma\Omega_{ro}$ is the variation in rotor speed, x_{fa} and $\dot{x}_{fa}$ are the fore-aft displacement and velocity respectively, x_{sw} and $\dot{x}_{sw}$ are the sideward displacement and velocity respectively, ε and $\dot{\varepsilon}$ are the drive-train displacement and speed respectively, σv_i is the wind speed disturbance.

The constant state-space matrices A, D, G and the LPV state-space matrices B, C , F can be seen in [30], the aerodynamic constants are listed in [29].

The identification step is given to verify the model accuracy. The parameters are defined as follows, The sample time $t = 0.1$ s, the samples $N = 1000$, the period $p = 10$, and the window size $d = 10$. The eigenvalues of $A^{(i)}$ matrices of the wind turbine are shown in Fig. 2.

It can be seen as a satisfactory model. To test the cross validation, the standard variance-accounted-for(VAF) is used. The VAF is often used to verify the correctness of a model, by comparing the real output with the estimated output of the model [31]. The VAF is defined as

$$\mathrm{VAF} = \max\{1 - \frac{\mathrm{var}(y_k - \hat{y}_k)}{\mathrm{var}(y_k)}, 0\} \times 100, \tag{29}$$

where y_k and $\hat{y}_k$ are the values at instant k of process and model output respectively.

Table 2. The VAF of LPV and LTI identified model.

Identified method	LPV	LTI
$\overline{\text{VAF}}$	86.1013	62.0634

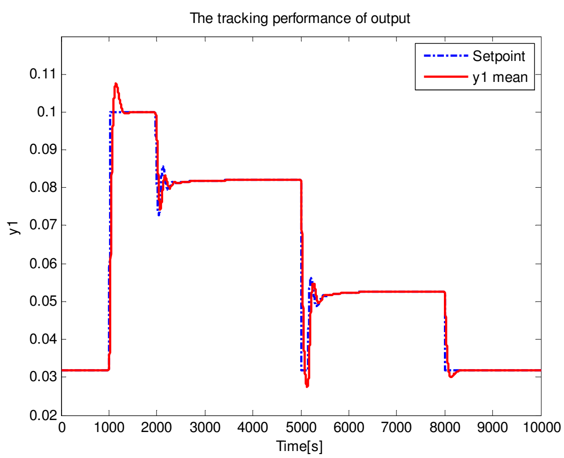

Fig. 3. The tracking performance of y_1.

The average $\overline{\text{VAF}}$ of VAF can be represented as

$$\overline{\text{VAF}} = \frac{\sum_{k=1}^{m} \text{VAF}_k}{m}. \tag{30}$$

The LTI(Linear Time Invariant) method of subspace identification in [32] is introduced as a comparison. The VAF on the validation data set can be seen in Table 2. We can get from (29) that with the increase of VAF, the model is more satisfactory. The cross validation results indicate that the LPV identified model is more accurate than LTI identified model.

Then, the identified LPV model is used to design the continuous-time data-driven predictive controller. The parameters of the controller are tuned as follows. The prediction horizon $N_2 = 30$ and the control horizon $N_u = 20$. The weighting $\lambda = 0.2$. Take the first 10000s for simulation validation. There are six outputs in this system. In the setpoint test, the performance of outputs using the proposed control method expresses satisfactory results as shown in Figs. 3-8.

For comparison, the LTI data-driven predictive controller(LTI-DPC) in [33] is used to control this system. Here 15000s is conducted, in the y_4 tracking test, Rf is defined as the reference output. The parameters of this controller are same as above proposed controller(we just call it LPV-DPC). As shown in Fig. 9, from about 3001s to 13000s, since the nonlinear character, the y_4 with the LTI-DPC can not achieve tracking the reference setpoint well. For the sake of clarity, a form of prediction error ξ

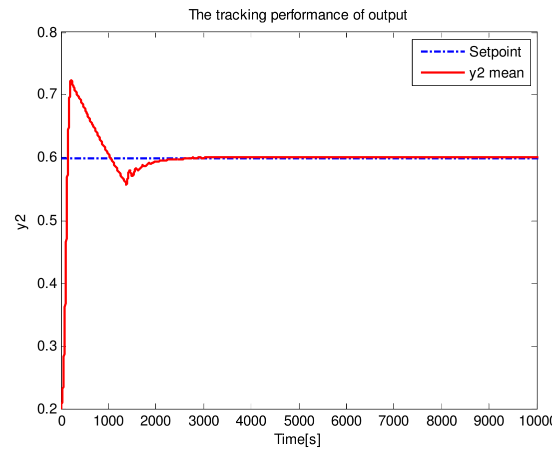

Fig. 4. The tracking performance of y_2.

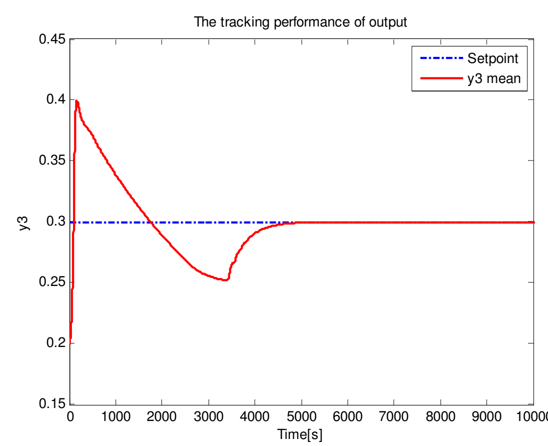

Fig. 5. The tracking performance of y_3.

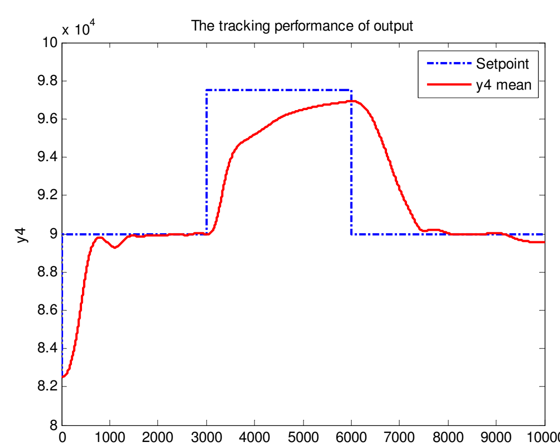

Fig. 6. The tracking performance of y_4.

is conducted to verify the performance of output:

$$\xi = \sqrt{\frac{\sum_{i=1}^{N} (y_i - y_i^p)^2}{\sum_{i=1}^{N} (y_i)^2}} * 100, \tag{31}$$

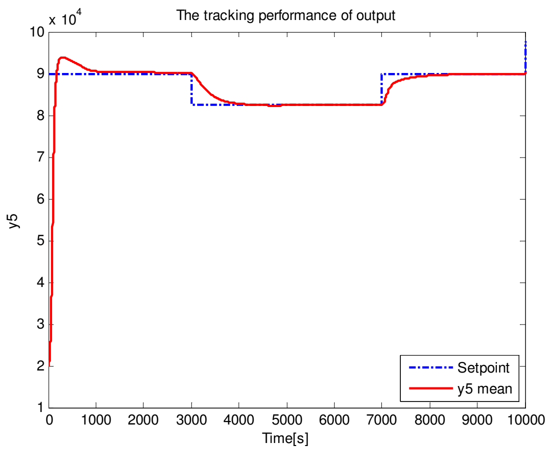

Fig. 7. The tracking performance of y_5.

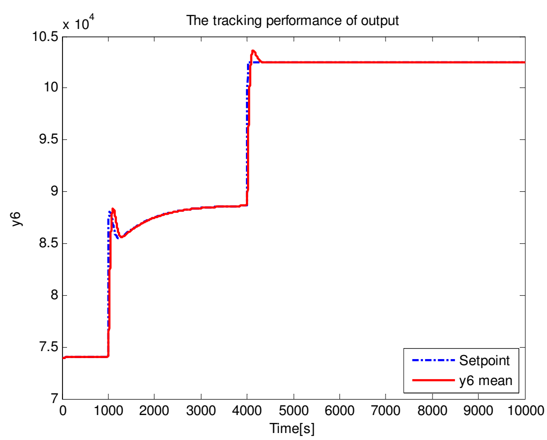

Fig. 8. The tracking performance of y_6.

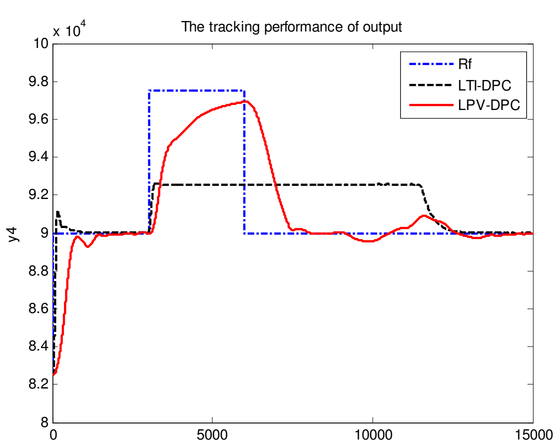

Fig. 9. The response comparison of y_4 tracking performance.

where y_i and y_i^p are the values at instant i of reference and process output respectively. We get the results of Table 3 below from Fig. 9. The performance of output using the LPV-DPC expresses better comparing with the LTI-DPC.

Table 3. The prediction error of LTI-DPC and LPV-DPC methods.

Control method	LTI-DPC	LPV-DPC
Prediction error ξ	3.2434	2.4416

5. CONCLUSIONS

In the article, we presented a continuous-time subspace based data-driven predictive control method for linear parameter varying(LPV) systems. The time varying matrices and Laguerre filters are used to get the subspace prediction of output through recursive substitution. Then, the data-driven predictive controller applied to LPV systems is designed using the subspace predictors which can be obtained from subspace prediction of output. The method was implemented on a wind turbine example to verify the effectiveness.

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